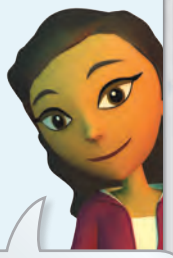
4-5 Isosceles and Equilateral Triangles

Mathematics Florida Standards

MAFS.912.G-CO.3.10 Prove theorems about triangles. Also MAFS.912.G-CO.3.11, MAFS.912.G-CO.4.12, MAFS.912.G-SRT.2.5

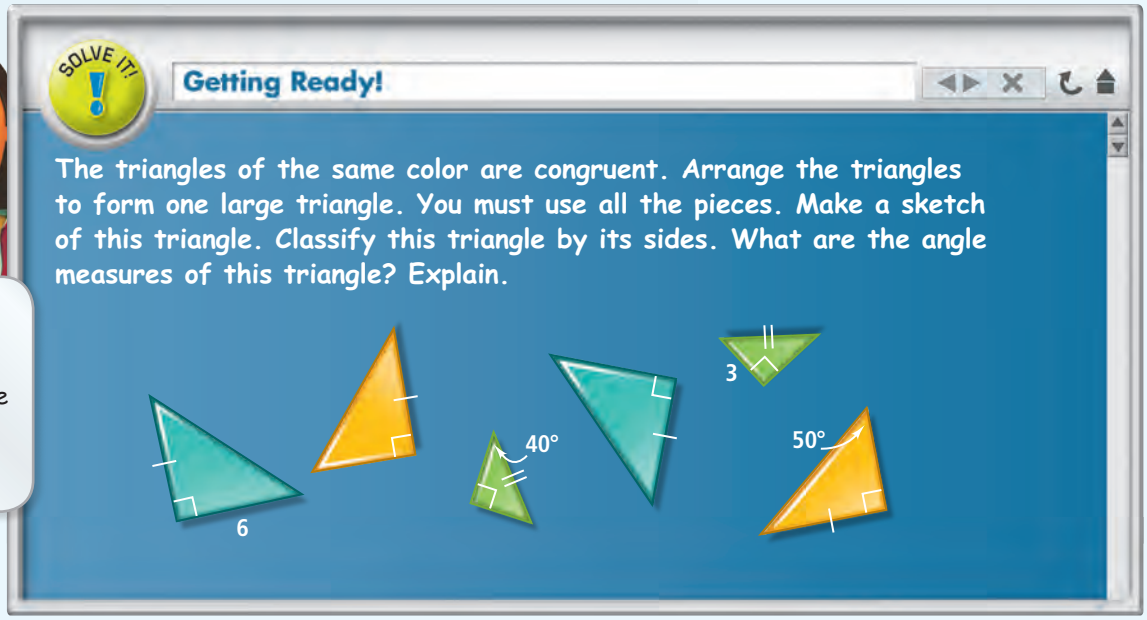
MP 1, MP 3, MP 4

Objective To use and apply properties of isosceles and equilateral triangles



Solving puzzles is fun! Work with the pieces until they make a whole triangle. Look for patterns in your solution.

MATHEMATICAL PRACTICES

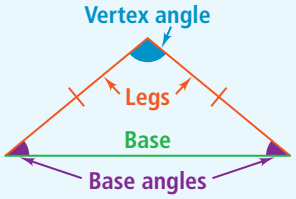


In the Solve It, you classified a triangle based on the lengths of its sides. You can also identify certain triangles based on information about their angles. In this lesson, you will learn how to use and apply properties of isosceles and equilateral triangles.

Essential Understanding The angles and sides of isosceles and equilateral triangles have special relationships.

Lesson Vocabulary
- legs of an isosceles triangle
- base of an isosceles triangle
- vertex angle of an isosceles triangle
- base angles of an isosceles triangle
- corollary

Isosceles triangles are common in the real world. You can frequently see them in structures such as bridges and buildings, as well as in art and design. The congruent sides of an isosceles triangle are its **legs.** The third side is the **base.** The two congruent legs form the **vertex angle.** The other two angles are the **base angles.**

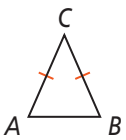


take note

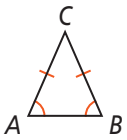
Theorem 4-3 Isosceles Triangle Theorem

Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

If . . .
$\overline{AC} \cong \overline{BC}$

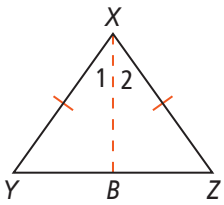


Then . . .
$\angle A \cong \angle B$



The proof of the Isosceles Triangle Theorem requires an auxiliary line.

Proof Proof of Theorem 4-3: Isosceles Triangle Theorem



Begin with isosceles $\triangle XYZ$ with $\overline{XY} \cong \overline{XZ}$. Draw $\overline{XB}$, the bisector of the vertex angle $\angle YXZ$.

Given: $\overline{XY} \cong \overline{XZ}$, $\overline{XB}$ bisects $\angle YXZ$

Prove: $\angle Y \cong \angle Z$

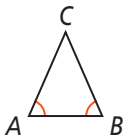
Proof: $\overline{XY} \cong \overline{XZ}$ is given. By the definition of angle bisector, $\angle 1 \cong \angle 2$. By the Reflexive Property of Congruence, $\overline{XB} \cong \overline{XB}$. So by the SAS Postulate, $\triangle XYB \cong \triangle XZB$. $\angle Y \cong \angle Z$ since corresponding parts of congruent triangles are congruent.

take note

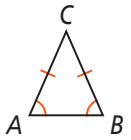
Theorem 4-4 Converse of the Isosceles Triangle Theorem

Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

If . . .
$\angle A \cong \angle B$



Then . . .
$\overline{AC} \cong \overline{BC}$



You will prove Theorem 4-4 in Exercise 23.

Problem 1 Using the Isosceles Triangle Theorems

Think

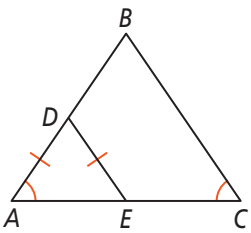
What are you looking for in the diagram?
To use the Isosceles Triangle Theorems, you need a pair of congruent angles or a pair of congruent sides.

A **Is $\overline{AB}$ congruent to $\overline{CB}$? Explain.**

Yes. Since $\angle C \cong \angle A$, $\overline{AB} \cong \overline{CB}$ by the Converse of the Isosceles Triangle Theorem.

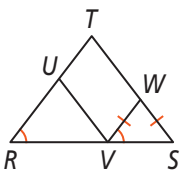
B **Is $\angle A$ congruent to $\angle DEA$? Explain.**

Yes. Since $\overline{AD} \cong \overline{ED}$, $\angle A \cong \angle DEA$ by the Isosceles Triangle Theorem.



Got It? **1. a.** Is $\angle WVS$ congruent to $\angle S$? Is $\overline{TR}$ congruent to $\overline{TS}$? Explain.

b. Reasoning Can you conclude that $\triangle RUV$ is isosceles? Explain.



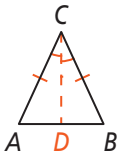
An isosceles triangle has a certain type of symmetry about a line through its vertex angle. The theorems in this lesson suggest this symmetry, which you will study in greater detail in Lesson 9-4.

take note

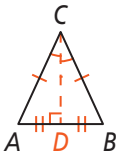
Theorem 4-5

Theorem
If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.

If . . .
$\overline{AC} \cong \overline{BC}$ and
$\angle ACD \cong \angle BCD$



Then . . .
$\overline{CD} \perp \overline{AB}$ and
$\overline{AD} \cong \overline{BD}$



You will prove Theorem 4-5 in Exercise 26.

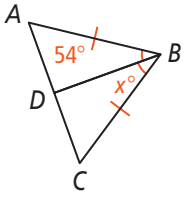
Think

What does the diagram tell you?
Since $\overline{AB} \cong \overline{CB}$, $\triangle ABC$ is isosceles. Since $\angle ABD \cong \angle CBD$, $\overline{BD}$ bisects the vertex angle of the isosceles triangle.

Problem 2 Using Algebra

GRIDDED RESPONSE

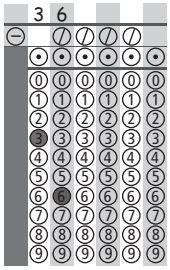
What is the value of x?



Since $\overline{AB} \cong \overline{CB}$, by the Isosceles Triangle Theorem, $\angle A \cong \angle C$. So $m\angle C = 54$.

Since $\overline{BD}$ bisects $\angle ABC$, you know by Theorem 4-5 that $\overline{BD} \perp \overline{AC}$. So $m\angle BDC = 90$.

$m\angle C + m\angle BDC + m\angle DBC = 180$	Triangle Angle-Sum Theorem
$54 + 90 + x = 180$	Substitute.
$x = 36$	Subtract 144 from each side.



Got It? **2.** Suppose $m\angle A = 27$. What is the value of x?

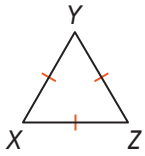
A **corollary** is a theorem that can be proved easily using another theorem. Since a corollary is a theorem, you can use it as a reason in a proof.

take note

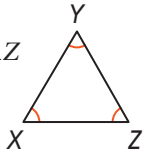
Corollary to Theorem 4-3

Corollary
If a triangle is equilateral, then the triangle is equiangular.

If . . .
$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



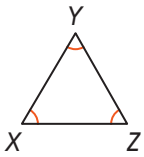
Then . . .
$\angle X \cong \angle Y \cong \angle Z$



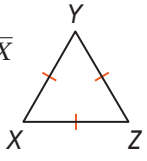
Corollary to Theorem 4-4

Corollary
If a triangle is equiangular, then the triangle is equilateral.

If . . .
$\angle X \cong \angle Y \cong \angle Z$

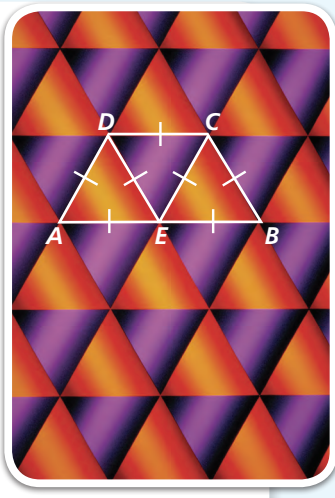


Then . . .
$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



Problem 3 Finding Angle Measures

Design What are the measures of $\angle A$, $\angle B$, and $\angle ADC$ in the photo at the right?



Think

The triangles are equilateral, so they are also equiangular. Find the measure of each angle of an equilateral triangle.

$\angle A$ and $\angle B$ are both angles in an equilateral triangle.

Use the Angle Addition Postulate to find the measure of $\angle ADC$.

Both $\angle ADE$ and $\angle CDE$ are angles in an equilateral triangle. So $m\angle ADE = 60$ and $m\angle CDE = 60$. Substitute into the above equation and simplify.

Write

Let a = measure of one angle.
$3a = 180$
$a = 60$

$m\angle A = m\angle B = 60$

$m\angle ADC = m\angle ADE + m\angle CDE$

$m\angle ADC = 60 + 60$
$m\angle ADC = 120$

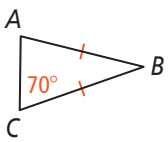
Got It? **3.** Suppose the triangles in Problem 3 are isosceles triangles, where $\angle ADE$, $\angle DEC$, and $\angle ECB$ are vertex angles. If the vertex angles each have a measure of 58, what are $m\angle A$ and $m\angle BCD$?

Lesson Check

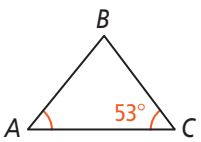
Do you know HOW?

1. What is $m\angle A$?

a.

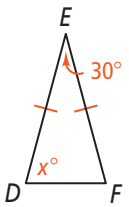


b.

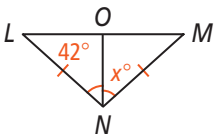


2. What is the value of x?

a.



b.

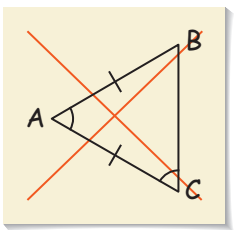


3. The measure of one base angle of an isosceles triangle is 23. What are the measures of the other two angles?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. What is the relationship between sides and angles for each type of triangle?
a. isosceles
b. equilateral

5. Error Analysis Claudia drew an isosceles triangle. She asked Sue to mark it. Explain why the marking of the diagram is incorrect.



Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Complete each statement. Explain why it is true.

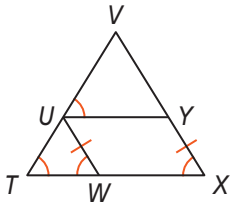
See Problem 1.

6. $\overline{VT} \cong$?

7. $\overline{UT} \cong$? $\cong \overline{YX}$

8. $\overline{VU} \cong$?

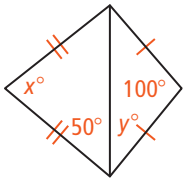
9. $\angle VYU \cong$?



Algebra **Find the values of x and y.**

See Problem 2.

10.



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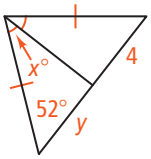

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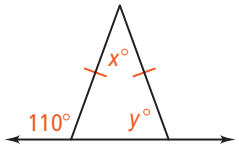
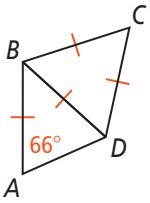


13. An equilateral triangle and an isosceles triangle share a common side. What is the measure of $\angle ABC$?

See Problem 3.



B Apply

STEM **14.** **Architecture** Each face of the Great Pyramid at Giza is an isosceles triangle with a 76° vertex angle. What are the measures of the base angles?

15. **Reasoning** What are the measures of the base angles of a right isosceles triangle? Explain.

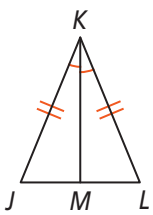
Given isosceles $\triangle JKL$ with base $\overline{JL}$, find each value.

16. If $m\angle L = 58$, then $m\angle LKJ =$? .

17. If $JL = 5$, then $ML =$? .

18. If $m\angle JKM = 48$, then $m\angle J =$? .

19. If $m\angle J = 55$, then $m\angle JKM =$? .



20. **Think About a Plan** A triangle has angle measures $x + 15$, $3x - 35$, and $4x$. What type of triangle is it? Be as specific as possible. Justify your answer.

- What do you know about the sum of the angle measures of a triangle?
- What do you need to know to classify a triangle?
- What type of triangle has no congruent angles? Two congruent angles? Three congruent angles?

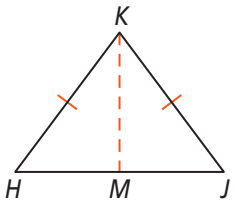
21. **Reasoning** An exterior angle of an isosceles triangle has measure 100. Find two possible sets of measures for the angles of the triangle.

22. Developing Proof Here is another way to prove the Isosceles Triangle Theorem. Supply the missing information.

Begin with isosceles $\triangle HKJ$ with $\overline{KH} \cong \overline{KJ}$.
Draw **a.** ? , a bisector of the base $\overline{HJ}$.

Given: $\overline{KH} \cong \overline{KJ}$, **b.** ? bisects $\overline{HJ}$

Prove: $\angle H \cong \angle J$



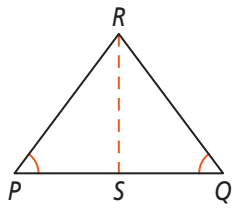
Statements	Reasons
1) $\overline{KM}$ bisects $\overline{HJ}$.	1) **c.** ?
2) $\overline{HM} \cong \overline{JM}$	2) **d.** ?
3) $\overline{KH} \cong \overline{KJ}$	3) Given
4) $\overline{KM} \cong \overline{KM}$	4) **e.** ?
5) $\triangle KHM \cong \triangle KJM$	5) **f.** ?
6) $\angle H \cong \angle J$	6) **g.** ?

23. Proof Supply the missing information in this statement of the Converse of the Isosceles Triangle Theorem. Then write a proof.

Begin with $\triangle PRQ$ with $\angle P \cong \angle Q$.
Draw **a.** ? , the bisector of $\angle PRQ$.

Given: $\angle P \cong \angle Q$, **b.** ? bisects $\angle PRQ$

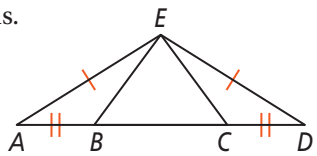
Prove: $\overline{PR} \cong \overline{QR}$



24. Writing Explain how the corollaries to the Isosceles Triangle Theorem and its converse follow from the theorems.

25. Proof **Given:** $\overline{AE} \cong \overline{DE}$, $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABE \cong \triangle DCE$

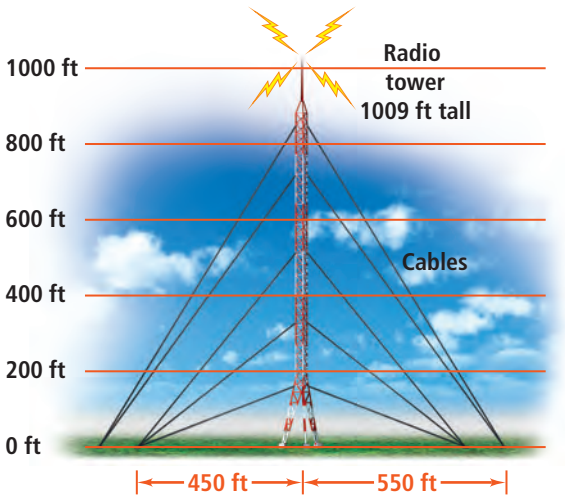


26. Prove Theorem 4-5. Use the diagram next to it on page 252.

27. Proof STEM **a. Communications** In the diagram at the right, what type of triangle is formed by the cables of the same height and the ground?

b. What are the two different base lengths of the triangles?

c. How is the tower related to each of the triangles?

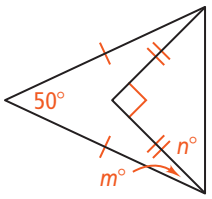


28. Algebra The length of the base of an isosceles triangle is x. The length of a leg is $2x - 5$. The perimeter of the triangle is 20. Find x.

29. Constructions Construct equilateral triangle ABC. Justify your method.

Algebra Find the values of m and n.

30.



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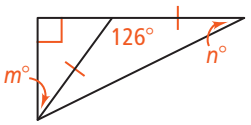

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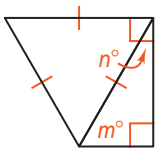


Challenge **Coordinate Geometry** **For each pair of points, there are six points that could be the third vertex of an isosceles right triangle. Find the coordinates of each point.**

33. (4, 0) and (0, 4)

34. (0, 0) and (5, 5)

35. (2, 3) and (5, 6)

36. Reasoning What measures are possible for the base angles of an acute isosceles triangle?

Standardized Test Prep

SAT/ACT

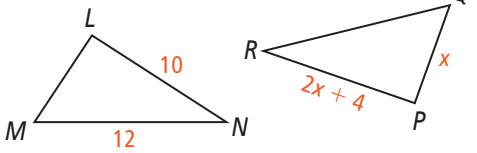
37. In isosceles $\triangle ABC$, the vertex angle is $\angle A$. What can you prove?

(A) $AB = CB$ (B) $m\angle B = m\angle C$ (C) $\angle A \cong \angle B$ (D) $\overline{BC} \cong \overline{AC}$

38. $\triangle LMN \cong \triangle PQR$. What is LM?

(F) 3 (H) 8

(G) 4 (I) 10



39. What is the exact area of the base of a circular swimming pool with diameter 16 ft?

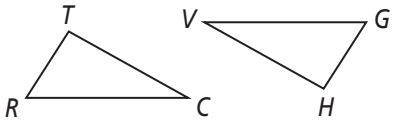
(A) 1018.29 ft^2 (B) 1018.3 ft^2 (C) 64π ft^2 (D) 256π ft^2

Short Response

40. Suppose $\triangle ABC$ and $\triangle DEF$ are nonright triangles. If $\angle B \cong \angle E$ and $\overline{AB} \cong \overline{DE}$, what else do you need to know to prove $\triangle ABC \cong \triangle DEF$? Explain.

Mixed Review

41. $m\angle R = 59$, $m\angle T = 93 = m\angle H$, $m\angle V = 28$, and $RT = GH$. What, if anything, can you conclude about RC and GV? Explain.



See Lesson 4-4.

42. Find the pattern of the sequence M, T, W, T, F, . . . Then find the next two terms.

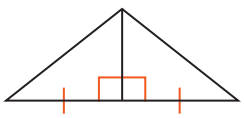
See Lesson 2-1.

Get Ready! **To prepare for Lesson 4-6, do Exercises 43 and 44.**

Can you conclude that the two triangles are congruent? Explain.

See Lesson 4-2.

43.



44.